M2a – Emergency Relief Systems

Also Source Estimation for ruptured pipelines or local failures in vessels

See also M2b for text and nomenclature
See also last slide for a special note on nomenclature

### Gas venting from ruptured pipelines

A Riemann Problem (a), Ruptured pipeline (b)

Initial conditions are known, and so is $a_i$

(a) $p_0, p_0, u_0 = 0 \quad p_1, p_1, u_1 = 0$

Choking condition $a_i = a_i$

(b) $p_0, p_0, u_0 = 0 \quad p_1, p_1, u_1$

$a_i, a_i$ are the speeds of sound in gas at corresponding states

Along the expansion wave $a_i = \frac{2}{y-1} u_i = \frac{2}{y-1} \frac{u_a}{\sqrt{\gamma}}$

At the sonic point $u_1 = a_i$. The sonic condition remains attached at the pipe exit

Then: $a_i = \left( \frac{\gamma-1}{\gamma} + 1 \right) u_i = \frac{\gamma+1}{\gamma} a_i$

Thus we can compute $a_i$

Also: $\frac{T_v}{\rho_v} \left( \frac{\alpha}{\rho_v} \right) - \left( \frac{\gamma}{\gamma+1} \right) \frac{p}{p_v} \left( \frac{\alpha}{\rho_v} \right)^{\gamma-1}$

### Gas venting from ruptured pipelines

**Critical Flow of Gases or Vapors**

When $P_e > 2 P_a$, the flow is choked (also called critical flow)

$G_{cr} = \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma}} \sqrt{\gamma P_e \rho_v} \sim \frac{1}{2} \sqrt{\gamma P_e \rho_v}$

$p_{cr} = \frac{P_e}{P_v} \gamma \sim 0.6$

Speed of sound at the exit plane.

$C_{cr} = \sqrt{\gamma RT_{cr}} = \sqrt{\frac{P_e}{\rho_{cr}}} R$ is the gas constant

8,314 m$^3$ s$^{-2}$ K$^{-1}$ kg$^{-1}$ mole$^{-1}$

Note that you have to divide by the molecular weight of the gas

### Vapor venting from vessels through nozzles

**Vapor Only or Two-Phase Vent?**

$U_\infty = 1.53 \left( \frac{\alpha g \rho_f - \rho_g}{\rho_f - \rho_f} \right)^{1/4} U = (1 - \alpha)^{-1} U_\infty$

Freeboard volume

$\alpha = \frac{H_e - H_f}{H_e} = 1 - \frac{H_e}{H_v}$

$\bar{\alpha} = 1 - \frac{H_v}{H_{2v}}$

if $\bar{\alpha} < \alpha$ $H_{2v} < H_v$ vapor venting

if $\bar{\alpha} = \alpha$ $H_{2v} = H_v$ two-phase venting

### Vapor venting from vessels through nozzles

**ARRESTING the RUNAWAY UNDER TWO-PHASE VENT**

$\dot{Q}_T = h_{f, g \rho g} (A J_{g0} + A J_{f2})$

Define: $J_{f2} \rho_f = G_f = (1 - \chi) G$

$\chi$ is the mixture quality

$\dot{Q}_T = h_{f, g \rho g} \left( \frac{J_{g2} \rho_g}{\rho_f} + \frac{J_{f2} \rho_f}{\rho_f} \right) A$

$= h_{f, g \rho g} \left( \frac{\chi G}{\rho_f} + \left( 1 - \frac{\chi G}{\rho_f} \right) \right) A$

$= h_{f, g \rho g} \left( \frac{\chi G + 1 - \chi G}{\rho_f} \right) G A$

So we need $G$
SOME USEFUL RELATIONSHIPS

To get $G$ critical maximize with respect to

exit condition (P) under various assumptions.

H-E and S-E are the most common (see Figs 1-4).

A rather general result:

When the fluid in the vent is nearly all saturated liquid

the equilibrium rate model provides a good approximation:

$$G = \frac{\rho U}{\rho} = \frac{1}{2} \left( h_f - h \right)$$

The ratio $h/v$ can be obtained

from the Clausius -Clapeyron

$$h = h_0 + \frac{1}{2} U^2$$

$$h = h_0 + \frac{\rho}{\rho_0}$$

$$\frac{\rho}{\rho_0} \sim 0.8$$

We refer to Figs 1-4 for details.

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**HOMOGENEOUS-EQUILIBRIUM CRITICAL FLOW**

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Fig.1 Critical mass flux according to Homogeneous-Equilibrium Model

H-E Model

Above this line the fluid is

subcooled liquid.

Below it is two-phase.

Use this plot to show how
tempering capacity increases with
vapor content entering the vent.

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Fig.2 Critical pressure according to H-E Model

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Fig.3 Critical mass flux according to the Slip-Equilibrium Model

S-E Model

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Fig.4 Critical pressure according to S-E Model

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**WATER**
The Omega Method

\[ \alpha_b = \alpha_i + (1 - \alpha_i) \rho_i C_p \frac{T_i}{T_f} \left( \frac{u_b}{u_f} \right)^2 \] and Figure 5

**When** \( \alpha_b > 4 \) **we have the analytical form:**

\[ \frac{G_{c,2p}}{u_b} = \frac{1}{\sqrt{\alpha_b}} \left( 0.6055 - 0.1356 \ln \alpha_b - 0.0131 (\ln \alpha_b)^2 \right) \]

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**Allowing for Disengagement**

**Finding out the level swell at a given venting rate:**

\[ J_{\text{ex}} = \frac{Q_{\text{ex}}}{h_{2p} \rho_{\text{ex}}} A_e = \frac{\dot{Q}}{h_{2p} \rho_{\text{ex}} A_e} \]

\[ \psi = \frac{J_{\text{ex}}}{U_{\text{ex}}} \]

**Churn Flow Regime**

\[ J_{\text{ex}}/U_{\text{ex}} = \psi = \frac{2\pi}{1 - \alpha} \]

**Bubbly Flow Regime**

\[ J_{\text{ex}}/U_{\text{ex}} = \psi = \frac{\pi (1 - \pi^2)}{1 - 1.2\pi} \]

Solve for average void fraction and compare to initial void fraction

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**Derivation in page 11 of pdf handout on ERS's**

\[ \frac{d\alpha}{(1-\alpha)^2} = \frac{\dot{Q}}{h_{\text{ex}}} (1-\alpha) H_{2p} dH = \frac{J_{\text{ex}}}{H_{2p}} dH = \psi \frac{dH}{H_{2p}} = \psi dH' \]

Which can be readily integrated to yield:

\[ \alpha = \frac{\psi}{1 + \psi} \]

And moreover, with another integration:

\[ \sigma = \frac{1}{\psi} \left( \alpha (H') dH' \right) \]

**Note:** the liquid density appears above because we defined the power density as per unit mass of liquid. \( Q_{\text{ex}} \) (no subscript) is per unit volume, and if used it would not need the liquid density. It is important to always check the units and understand the meaning of equations.

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The maximum void fraction is at the top of the two-phase mixture is obtained by setting \( H = 1 \):

\[ \alpha_{\text{max}} = \frac{\psi}{1 + \psi} \]

And with the average void fraction expression found above, we also have:

\[ \alpha_{\text{max}} = \frac{2\pi}{1 + 2\pi} \]

Such estimates allow us to judge if and how to increase the quality of the two-phase mixture entering the vent (less liquid), and thus reduce the size of the vent.
Accounting for Overpressure

\[ \Delta P = P_{\text{max}} - P_s \] then obtain corresponding \( \Delta T \)

\[ \frac{A_g}{A} = \left(1 + \rho f(\psi - \rho_f)\right)^2 \]

\[ \psi = \frac{C_\rho \Delta T \left(\alpha + \frac{\psi - \rho_f}{\rho_f}\right)}{h_f} + \frac{C_\rho \Delta T \left(1 - \alpha - \frac{\psi - \rho_f}{\rho_f}\right)}{h_f} \]

Isentropic Expansions

For precise calculations we need the "steam table" for the liquid considered.

When not available we find the Antoine constants, and thus have the vapor pressure information. Then the enthalpy of evaporation can be obtained from the Clausius-Clapeyron equation (which gives us the ratio with specific volume).

Then the amount of liquid vaporized can be obtained approximately from an energy balance—the sensible heat of liquid between its initial temperature and the saturation temperature at the final pressure to which the expansion reached, is utilized to provide the latent heat of evaporation:

\[ Q = m C_{\rho} (T_L - T^*(P_f)) \]

\[ \chi = \frac{m}{m} \frac{C_{\rho} (T_L - T^*(P_f))}{h_f} \]

Note on nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>( \hat{Q} )</td>
<td>Heat generation rate per unit volume of reaction mixture</td>
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<tr>
<td>( \hat{Q} _T )</td>
<td>Total heat generation rate in vessel</td>
</tr>
<tr>
<td>( H_s )</td>
<td>Height of vessel</td>
</tr>
<tr>
<td>( H_L )</td>
<td>Height of liquid level in vessel</td>
</tr>
<tr>
<td>( P_S )</td>
<td>Set pressure of ER actuation</td>
</tr>
<tr>
<td>( P_0 )</td>
<td>Stagnation pressure in vessel</td>
</tr>
<tr>
<td>( \chi )</td>
<td>Static quality</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Quality (a flow quantity)</td>
</tr>
<tr>
<td>( C )</td>
<td>Speed of sound in gas, or liquid or two-phase mixture</td>
</tr>
</tbody>
</table>

\[ \sigma = \frac{\psi}{2 + \psi} \]

\[ \alpha_{\text{max}} = \frac{\psi}{1 + \psi} \]