Dynamic Behavior

In analyzing process dynamic and process control systems, it is important to know how the process responds to changes in the process inputs.

A number of standard types of input changes are widely used for two reasons:

1. They are representative of the types of changes that occur in plants.

2. They are easy to analyze mathematically.

1. Step Input

A sudden change in a process variable can be approximated by a step change of magnitude, M:

$$U_s \triangleq \begin{cases} 0 & t < 0 \\ M & t \geq 0 \end{cases} \qquad (5\text{-}4)$$

The step change occurs at an arbitrary time denoted as $t = 0$.

- *Special Case:* If $M = 1$, we have a "unit step change". We give it the symbol, $S(t)$.

- *Example of a step change:* A reactor feedstock is suddenly switched from one supply to another, causing sudden changes in feed concentration, flow, etc.

Example:

The heat input to the stirred-tank heating system in Chapter 2 is suddenly changed from 8000 to 10,000 kcal/hr by changing the electrical signal to the heater. Thus,

$$Q(t) = 8000 + 2000S(t), \qquad S(t) \triangleq \text{unit step}$$

and

$$Q'(t) = Q - \bar{Q} = 2000S(t), \qquad \bar{Q} = 8000 \text{ kcal/hr}$$

2. Ramp Input

- Industrial processes often experience "drifting disturbances", that is, relatively slow changes up or down for some period of time.
- The rate of change is approximately constant.

We can approximate a drifting disturbance by a *ramp input:*

$$U_R(t) \triangleq \begin{cases} 0 & t < 0 \\ at & t \geq 0 \end{cases} \qquad (5\text{-}7)$$

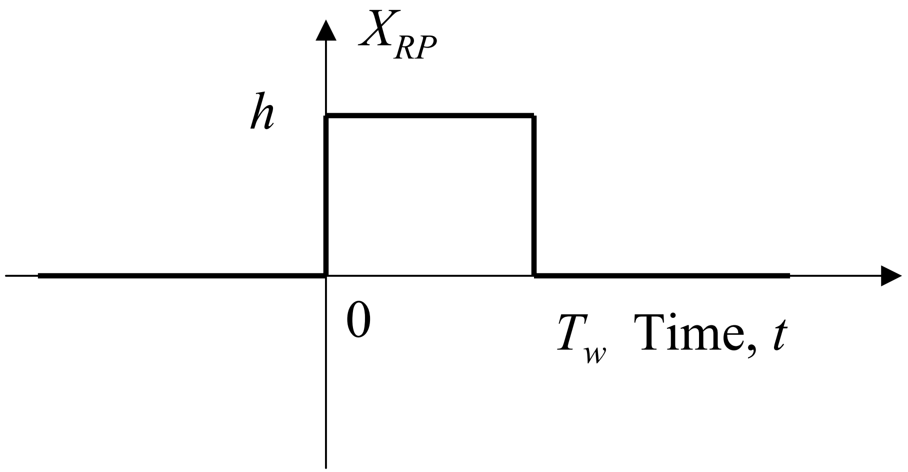
Examples of ramp changes:

1. Ramp a setpoint to a new value. (Why not make a step change?)
2. Feed composition, heat exchanger fouling, catalyst activity, ambient temperature.

3. Rectangular Pulse

It represents a brief, sudden change in a process variable:

$$U_{RP}(t) \triangleq \begin{cases} 0 & \text{for } t < 0 \\ h & \text{for } 0 \le t < t_w \\ 0 & \text{for } t \ge t_w \end{cases} \tag{5-9}$$



Examples:

1. Reactor feed is shut off for one hour.
2. The fuel gas supply to a furnace is briefly interrupted.

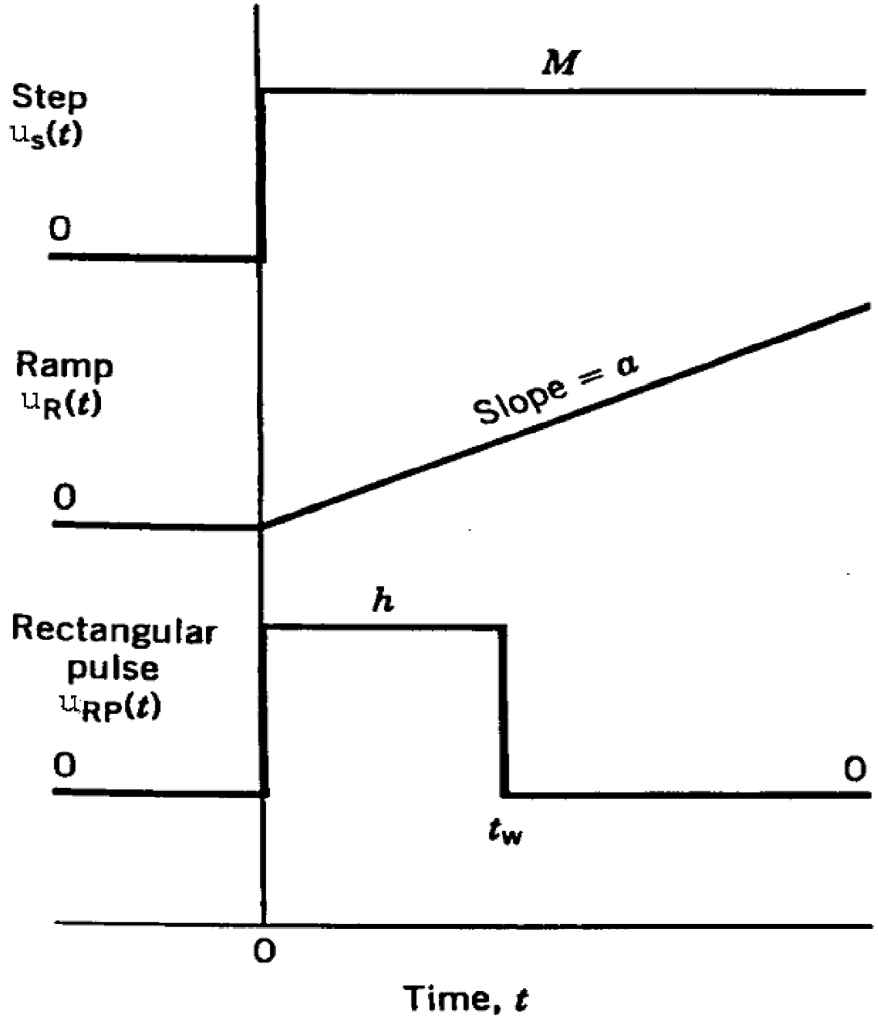


Figure 5.2. Three important examples of deterministic inputs.

Chapter 5

4. Sinusoidal Input

Processes are also subject to periodic, or cyclic, disturbances. They can be approximated by a sinusoidal disturbance:

$$U_{\sin}(t) \triangleq \begin{cases} 0 & \text{for} \quad t < 0 \\ A\sin(\omega t) & \text{for} \quad t \geq 0 \end{cases} \tag{5-14}$$

where: A = amplitude, ω = angular frequency

Examples:

1. 24 hour variations in cooling water temperature.
2. 60-Hz electrical noise (in the USA)

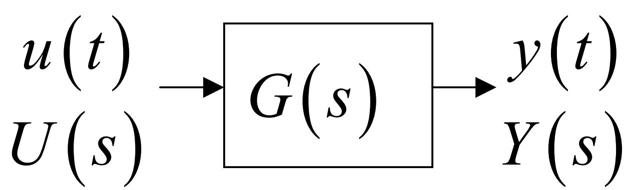
5. Impulse Input

- Here, $U_I(t) = \delta(t)$.
- It represents a short, transient disturbance.

Examples:

1. Electrical noise spike in a thermo-couple reading.
2. Injection of a tracer dye.

- Useful for analysis since the response to an impulse input is the inverse of the TF. Thus,



Here,

$$Y(s) = G(s)U(s) \qquad (1)$$

The corresponding time domain express is:

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau \qquad (2)$$

where:

$$g(t) \triangleq \mathfrak{L}^{-1}\left[G(s)\right] \qquad (3)$$

Suppose $u(t) = \delta(t)$. Then it can be shown that:

$$y(t) = g(t) \qquad (4)$$

Consequently, $g(t)$ is called the "impulse response function".

First-Order System

The standard form for a first-order TF is:

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} \qquad (5\text{-}16)$$

where:

$$K \triangleq \text{steady-state gain}$$

$$\tau \triangleq \text{time constant}$$

Consider the response of this system to a step of magnitude, M:

$$U(t) = M \text{ for } t \geq 0 \qquad \Rightarrow U(s) = \frac{M}{s}$$

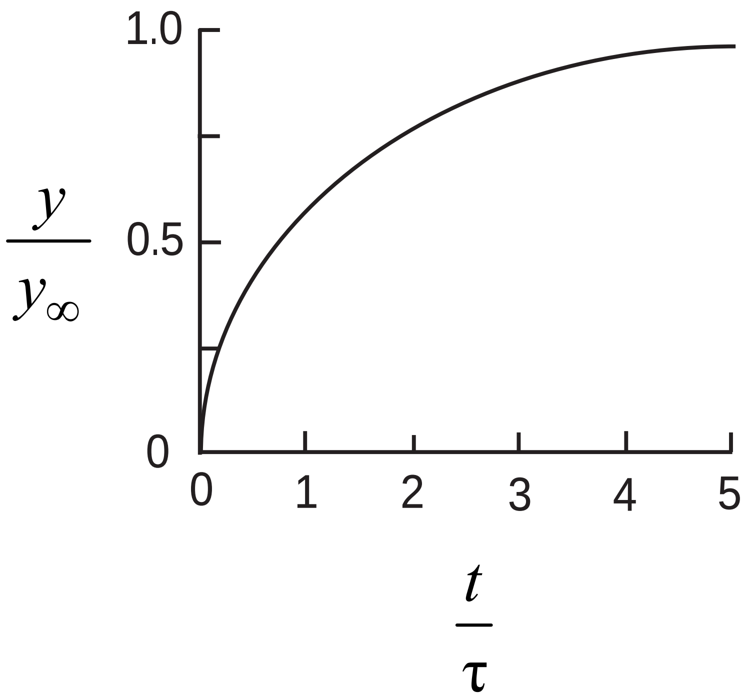
Substitute into (5-16) and rearrange,

$$Y(s) = \frac{KM}{s(\tau s + 1)} \qquad (5\text{-}17)$$

Take $\mathfrak{L}^{-1}$ (cf. Table 3.1),

$$\boxed{y(t) = KM\left(1 - e^{-t/\tau}\right)} \qquad (5\text{-}18)$$

Let $y_\infty \triangleq$ steady-state value of $y(t)$. From (5-18), $y_\infty = KM$.



t	$\frac{y}{y_\infty}$
0	0
τ	0.632
2τ	0.865
3τ	0.950
4τ	0.982
5τ	0.993

Note: Large τ means a slow response.

Integrating Process

Not all processes have a steady-state gain. For example, an "integrating process" or "integrator" has the transfer function:

$$\frac{Y(s)}{U(s)} = \frac{K}{s} \qquad (K = \text{constant})$$

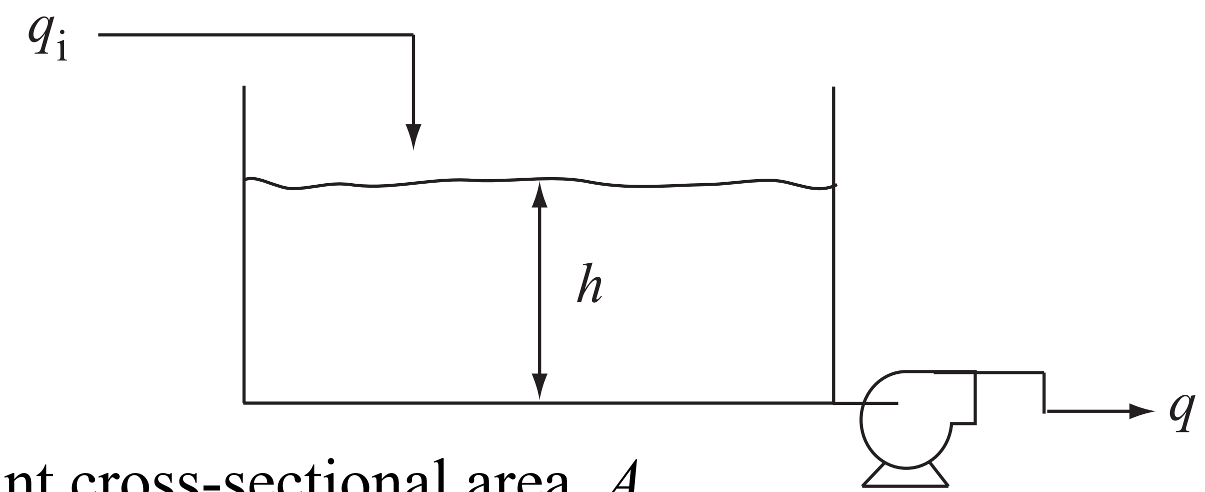
Consider a step change of magnitude M. Then $U(s) = M/s$ and,

$$Y(s) = \frac{KM}{s^2} \overset{\mathfrak{L}^{-1}}{\Rightarrow} y(t) = KMt$$

Thus, $y(t)$ is unbounded and a new steady-state value does *not* exist.

Common Physical Example:

Consider a liquid storage tank with a pump on the exit line:



- Assume:
  1. Constant cross-sectional area, A.
  2. $q \neq f(h)$
- Mass balance: $A\dfrac{dh}{dt} = q_i - q$ (1) $\Rightarrow$ $0 = \bar{q}_i - \bar{q}$ (2)
- Eq. (1) – Eq. (2), take $\mathfrak{L}$, assume steady state initially,

$$H'(s) = \frac{1}{As}\left[Q_i'(s) - Q'(s)\right]$$

- For $Q'(s) = 0$ (constant q),

$$\boxed{\frac{H'(s)}{Q_i'(s)} = \frac{1}{As}}$$

Second-Order Systems

- Standard form:

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1} \qquad (5\text{-}40)$$

which has three model parameters:

$$K \triangleq \text{steady-state gain}$$

$$\tau \triangleq \text{"time constant" [=] time}$$

$$\zeta \triangleq \text{damping coefficient (dimensionless)}$$

- Equivalent form: $\left(\omega_n \triangleq \text{natural frequency} = \frac{1}{\tau}\right)$

$$\frac{Y(s)}{U(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- The type of behavior that occurs depends on the numerical value of damping coefficient, ζ :

It is convenient to consider three types of behavior:

Damping Coefficient	Type of Response	Roots of Charact. Polynomial
$\zeta > 1$	Overdamped	Real and $\neq$
$\zeta = 1$	Critically damped	Real and $=$
$0 \leq \zeta < 1$	Underdamped	Complex conjugates

- Note: The characteristic polynomial is the denominator of the transfer function:

$$\tau^2 s^2 + 2\zeta\tau s + 1$$

- What about $\zeta < 0$? It results in an unstable system

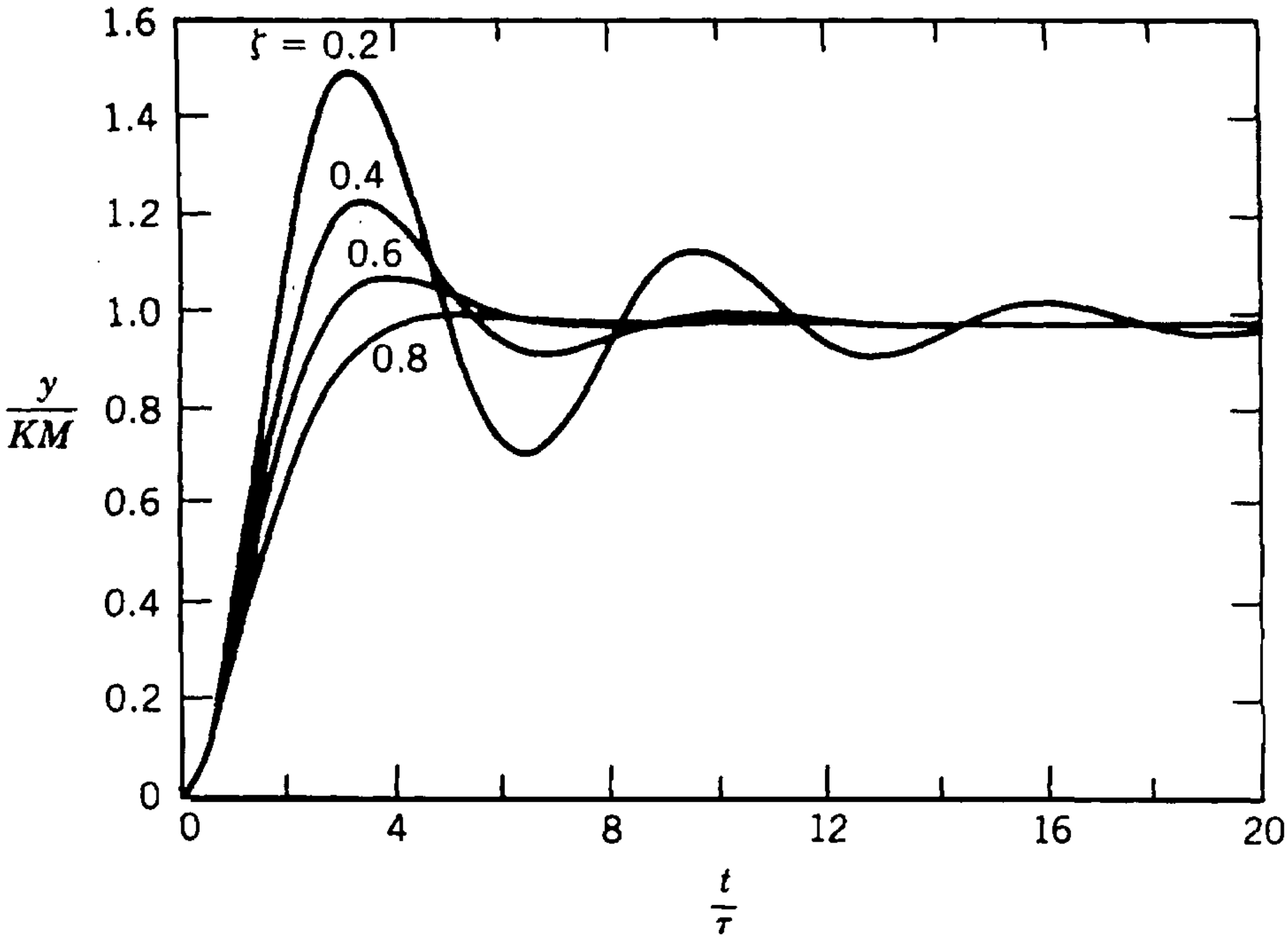


Figure 5.8. Step response of underdamped second-order processes.

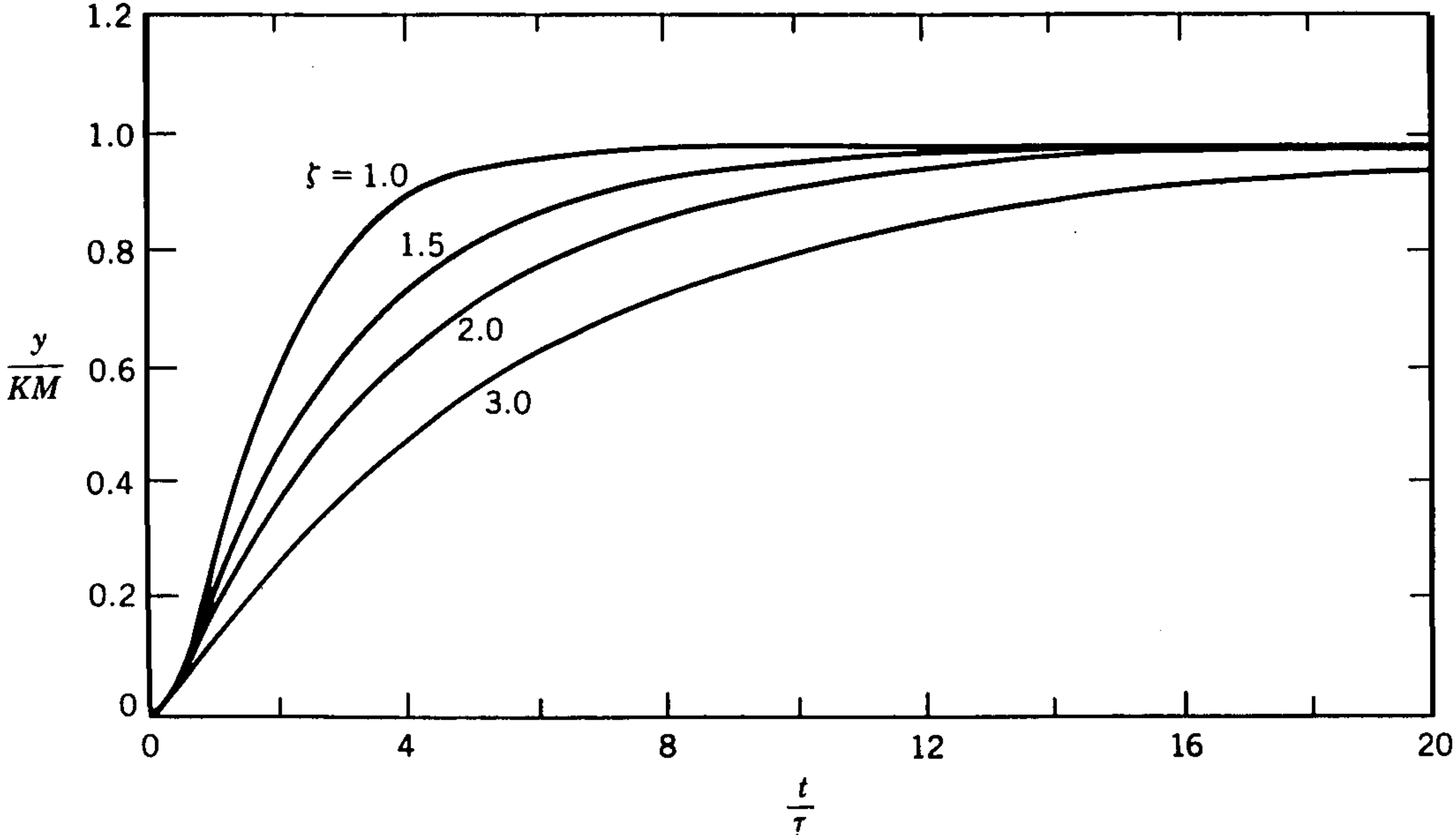


Figure 5.9. Step response of critically-damped and overdamped second-order processes.

Several general remarks can be made concerning the responses show in Figs. 5.8 and 5.9:

1. Responses exhibiting oscillation and overshoot ($y/KM > 1$) are obtained only for values of ζ less than one.
2. Large values of ζ yield a sluggish (slow) response.
3. The fastest response without overshoot is obtained for the critically damped case $(\zeta = 1)$.

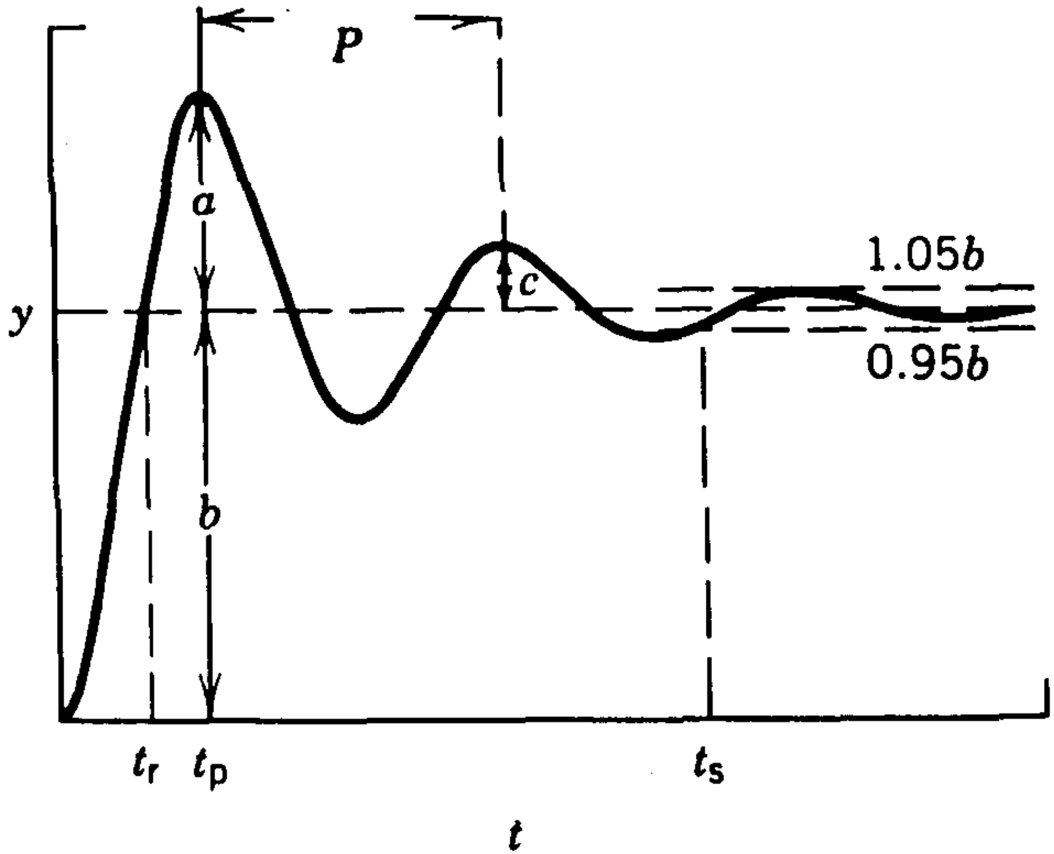


Figure 5.10. Performance characteristics for the step response of an underdamped process.

1. **Rise Time:** t_r is the time the process output takes to first reach the new steady-state value.

2. **Time to First Peak:** t_p is the time required for the output to reach its first maximum value.

3. **Settling Time**: t_s is defined as the time required for the process output to reach and remain inside a band whose width is equal to ±5% of the total change in y. The term 95% response time sometimes is used to refer to this case. Also, values of ±1% sometimes are used.

4. **Overshoot:** OS = a/b (% overshoot is $100a/b$).

5. **Decay Ratio**: DR = c/a (where c is the height of the second peak).

6. **Period of Oscillation**: P is the time between two successive peaks or two successive valleys of the response.