

Controller Performance Assessment Based on Setpoint Response Data

Anthony P. Swanda and Dale E. Seborg

Dept. of Chemical Engineering, Univ. of California, Santa Barbara, CA, 93106
swanda@sword.chemengr.ucsb.edu and seborg@engineering.ucsb.edu

Abstract: A new methodology is developed to assess the performance of PI controllers from closed-loop response data for a setpoint step change. It is based on two new dimensionless performance indices, the dimensionless settling time and the dimensionless integral of the absolute value of the error. The methodology is used to quantify how far a control loop is from the best achievable performance of PI control. It also identifies poorly performing control loops, such as those that are excessively sluggish. The performance and robustness properties of the Internal Model Control (IMC) tuning method are analyzed and are used as industrially relevant benchmarks. The proposed methodology is also applicable to PID controllers (Swanda, 1999).

1. INTRODUCTION

Despite the importance of process control, many industrial control loops operate poorly. In testing thousands of control loops in hundreds of operating plants, Techmation Inc. and others have found that more than 30% of the control loops actually *increase* variability over manual control due to poor controller tuning (Ender, 1993). In addition, pulp and paper mill auditing has shown that a large percentage of control loops actually de-stabilize product uniformity as a result of either valve stiction or poor controller tuning (Haggman and Bialkowski, 1992). One reason why so many control loops operate poorly is that there are often more than 1000 loops in large process plants and not enough control engineers to maintain every loop (Ender, 1993). Thus, there appears to be a strong need for automatic assessment and monitoring of control loop performance.

Assessment and monitoring of controller performance has been an active area of research for the last decade (Harris *et al.* 1999; Qin, 1998). The majority of work has focused on *stochastic performance monitoring* with minimum output variance as a performance benchmark, while less attention has been paid to traditional concerns of performance, such as the response to step changes in setpoint or disturbance variables.

Several time domain characteristics are traditionally used to describe a closed-loop response to a step change in setpoint: rise time, t_r , settling time, t_s , and overshoot, OS (Seborg *et al.*, 1989). The settling time is defined as the time the process output takes to reach and remain inside a band whose width is equal to $\pm \alpha$ of the difference between the initial and the final steady-state values. In this paper, we use $\alpha = 0.1$. Another characterization used to assess control

system performance is the integral of the absolute value of the error (IAE), where the error is the setpoint minus the output.

A first order plus time delay transfer function model is used to develop performance benchmarks,

$$\text{Model 1: } G(s) = \frac{K e^{-\theta_a s}}{\tau_a s + 1}$$

where the time delay θ_a will be referred to as the *apparent time delay*. It serves as a simple means to characterize the net effect that time delays, right half plane zeros, and process order have on the achievable performance. The t_s and IAE values are nondimensionalized using the apparent time delay. It will be shown that the dimensionless performance indices of a well-tuned controller are approximately independent of the model order of a linear system. Thus with limited knowledge of the underlying plant, the performance of the PI controller can be assessed.

In the IMC framework, the τ_c tuning parameter can be used to trade performance, in the terms of speed of response, for robustness. The trade-off is not optimal in the sense that some performance objective is minimized subject to a robustness constraint. However, this formulation allows a good trade-off between the conflicting objectives of performance and robustness. For PI control and Model 1, the Lambda Tuning formula given by Bialkowski (1996) and the Direct Synthesis formula derived by Seborg *et al.* (1989) are identical to the IMC formula given by Chien and Fruehauf (1990). Thus the following analysis is directly applicable to all three tuning methods.

There are several additional reasons to study the performance of the IMC formula. First, its popularity is growing; it is becoming widely used in the pulp and paper industry and chemical industry (Bialkowski, 1996; Fruehauf *et al.*, 1994). Second, the τ_c value can be selected to yield a setpoint step response with a near minimum IAE value for PI control (Rivera *et al.*, 1986), and as will be shown, a near minimum settling time.

2. DIMENSIONLESS PERFORMANCE INDICES

Åström *et al.* (1992) have related t_r values for Ziegler-Nichols tuned PID controllers to the apparent process time delay and have proposed using a dimensionless number, the normalized closed-loop rise time, τ , for evaluating closed-loop PID performance. Another indication of control system performance is the settling time, t_s , which can also

be nondimensionalized using the apparent time delay. Thus a dimensionless settling time is defined as

$$T_s \equiv \frac{t_s}{\theta_a} \quad (1)$$

where t_s is the settling time based on $\alpha = 0.1$. Benchmark values for T_s will be determined that are indicative of well-tuned controllers.

The settling time is based only on a single point of the response curve. Often it is important to characterize the entire response curve. One such characterization is the integral of the absolute value of the error, IAE. Shinsky (1990) argues that the IAE value is a good economic-performance measure because the size and length of the error in either direction is proportional to lost revenue. A dimensionless IAE index, IAE_d , for setpoint step changes is defined as

$$IAE_d \equiv \frac{IAE}{|r_o| \theta_a} \quad (2)$$

where r_o is the size of the setpoint step change. Benchmark values for IAE_d will be determined that are indicative of well-tuned controllers.

3. PERFORMANCE-ROBUSTNESS CHARACTERISTICS OF IMC-PI CONTROL

The performance-robustness characteristics of the IMC tuning method are analyzed in this section. The IMC-PI controller based on Model 1 is (Chien and Fruehauf, 1990)

$$G_c = \frac{(\tau_a s + 1)}{K(\tau_c + \theta_a)s} \quad (3)$$

The selection of tuning parameter τ_c depends on the control objective. Fruehauf *et al.* (1994) and (Rivera *et al.*, 1986) state that τ_c should be chosen greater than or equal to the (apparent) time delay. The minimum recommended τ_c value of θ_a results in a PI controller with a near minimum IAE value and little or no overshoot (Rivera *et al.*, 1986). In this analysis, only $\tau_c \geq \theta_a$ will be considered.

The closed-loop transfer function for Model 1 and the IMC-PI controller is

$$\frac{y}{r} = \frac{G_c G}{1 + G_c G} = \frac{e^{-\theta_a s}}{\left(\frac{\tau_c}{\theta_a} + 1\right) \theta_a s + e^{-\theta_a s}} \quad (4)$$

The closed-loop transfer function has several interesting properties. The response is independent of the process time constant, the shape of the response is given by τ_c/θ_a , and for a given τ_c/θ_a ratio, the entire response is determined by θ_a (Rivera *et al.*, 1986). Because the entire response is determined by θ_a for a given τ_c/θ_a value, response characteristics like settling time and IAE are also determined by θ_a .

An analytical expression for t_s can be derived using (4). A closed-loop transfer function can be determined by approximating the denominator time delay of (4) with a first order Taylor series. The resulting transfer function is

$$\frac{y}{r} = \frac{e^{-\theta_a s}}{\tau_c s + 1} \quad (5)$$

This approximation does not accurately describe the entire setpoint response curve well for small τ_c/θ_a values, but it can be used to estimate the settling time reasonably well (Swanda, 1999). The settling time is given by the time when $y(t)$, the time domain solution of (5), first reaches $0.9r_o$ where r_o is the size of the setpoint step change. Dividing the resulting t_s expression by θ_a gives

$$T_s = 2.30 \frac{\tau_c}{\theta_a} + 1 \quad (6)$$

Equation (6) is reasonably accurate because it overestimates the true dimensionless settling time by no more than a factor of 1.1 for $\tau_c/\theta_a \geq 1$ (Swanda, 1999). This difference approaches zero for large τ_c/θ_a values.

An analytical expression for IAE can also be derived using (4) and a 1/1 Padé approximation for the denominator time delay. The resulting closed-loop transfer function is

$$\frac{y}{r} = \frac{\frac{\theta_a}{2} s + 1}{\left(\frac{\tau_c}{\theta_a} + 1\right) \frac{\theta_a^2}{2} s^2 + \left(\frac{\tau_c}{\theta_a} + \frac{1}{2}\right) \theta_a s + 1} e^{-\theta_a s} \quad (7)$$

Only the solution for overdamped system will be considered because the resulting analytical expression is also accurate for slightly underdamped systems. The resulting expression for the dimensionless IAE value is

$$IAE_d = 1 + \frac{\tau_c}{\theta_a} \quad (8)$$

Equation (8) is quite accurate even for $\tau_c/\theta_a \geq 1$, which includes the both slightly underdamped, critically damped, and overdamped systems (Swanda, 1999).

Equation (6) can be combined with (8) to provide the following approximate relationship between T_s and IAE_d :

$$IAE_d = \frac{T_s}{2.30} + 0.566 \quad \text{for } T_s \geq 3.30 \quad (9)$$

The condition that $T_s \geq 3.3$ occurs because (6) and (8) are only valid for $\tau_c/\theta_a \geq 1$.

3.1 Performance-Robustness Trade-off

The open-loop transfer function model for Model 1 and the IMC-PI controller in (3) is

$$G_{OL} = G_c G = \frac{1}{\left(\tau_c + \frac{\theta_a}{2}\right)s} e^{-\theta_a s} \quad (10)$$

The corresponding gain margin, A_m , and phase margin, ϕ_m , can be expressed as a function of τ_c/θ_a (Swanda, 1999)

$$A_m \equiv \frac{1}{|G(j\omega_c)|} = \frac{\pi}{2} \left(\frac{\tau_c}{\theta_a} + 1 \right) \quad (11)$$

$$\phi_m \equiv \pi + \arg\{G_{OL}(j\omega_g)\} = \frac{\pi}{2} - \frac{1}{\frac{\tau_c}{\theta_a} + 1} \quad (12)$$

where ω_c is the phase crossover frequency and ω_g is the gain crossover frequency. Ho et al (1995) were the first to develop the relationship in (11). Equations (11) and (12) are simple and provide insight into how the τ_c tuning parameter relates to robustness. They can be used to determine the appropriate τ_c value for a desired gain-phase margin pair. Furthermore, they are also useful because they can be used to relate the gain and phase margins to the dimensionless performance indices. For example, solving (6) for τ_c/θ_a and substituting the result into (11) and (12) yields:

$$A_m = \frac{\pi}{2} \left(\frac{T_s}{2.30} + 0.565 \right) \quad (13)$$

$$\phi_m = \frac{\pi}{2} - \frac{1}{\frac{T_s}{2.30} + 0.565} \quad (14)$$

Because (6) is only applicable for $\tau_c/\theta_a \geq 1$, the above performance-robustness relationships are valid for $T_s \geq 3.3$.

3.2 Simulation Study

Equations (9), (13), and (14) are approximate and based on Model 1. It will be shown that they are accurate in spite of the approximation and that they are also applicable to other process models.

Closed-loop simulations were performed using Model 1 and four additional models:

$$\text{Model 2: } G(s) = \frac{e^{-s}}{(s+1)^2}$$

$$\text{Model 3: } G(s) = \frac{(1.25s+1)e^{-0.4s}}{(s+1)^2}$$

$$\text{Model 4: } G(s) = \frac{1}{(s+1)^5}$$

$$\text{Model 5: } G(s) = \frac{-0.75s+1}{(s+1)^3}$$

Note that Models 1 – 5 represent a wide range of open-loop stable processes. The standard form of the non-interacting PI controller was used (Seborg *et al.*, 1989),

$$\frac{U(s)}{E(s)} = K_c \left(1 + \frac{1}{\tau_I s} \right) \quad (15)$$

where U is the controller output, E is the error, K_c is the controller gain, and τ_I is the reset time.

Figure 1 compares the numerically determined T_s and IAE_d values for Models 1 – 5 with the analytical relationship (9). The T_s - IAE_d relationships are nearly the same for all five models. It also shows that there is a small difference between the data and (9) for small T_s values. To improve the accuracy of the relationship, linear regression was used to develop a straight line fit,

$$IAE_d = \frac{T_s}{2.43} + 0.878 \quad \text{for } T_s \geq 3.30 \quad (16)$$

which is also shown in Figure 1. Figures 2 and 3 show that the gain and phase margin relationships in (13) and (14) are in good agreement with the numerically determined values for Models 1 – 5. The simulation results in Figures 1 – 3 show that (9), (13), and (14) are accurate for a wide range of models and are not limited to Model 1.

4. OPTIMAL DIMENSIONLESS SETTLING TIME AND IAE VALUES

In this section, optimal T_s and IAE_d values are determined for Models 1-5, and serve as benchmarks for control system performance. Controller settings which minimize T_s and IAE_d values were determined using the MATLAB Optimization TOOLBOX (Grace, 1995).

Table 1 shows the optimal T_s and IAE_d values for Models 1 – 5. The optimal T_s and IAE_d values are almost exactly 2.3 and 2.0, respectively, and are insensitive to model order and model type. Note that the controller settings for optimal IAE are not the same as the controller settings for optimal settling time. The optimal values are industrially relevant performance benchmarks because the IMC-PI controller can approach them. Equations (6) and (8) show that for $\tau_c = \theta_a$, the T_s and IAE_d values are approximately 3.3 and 2.0, respectively.

5. PROPOSED METHODOLOGY FOR ASSESSING CONTROL LOOP PERFORMANCE

In order to quantify how far a PI controller is from the best achievable performance for PI control and to identify poorly performing control loops, three performance classes are defined: High Performance, Excessively Sluggish, and Poorly Tuned. Table 2 defines the three performance classes in terms of a bound for each dimensionless performance index. For a particular class, all conditions in Table 2 should be met. However, this definition can be relaxed to a single bound if one performance index is favored over another.

For the High Performance Class, the upper bound for T_s was chosen so that it is twice the optimal T_s value of 2.3. The IAE_d upper bound was chosen by substituting the T_s upper bound into (16). For a well-tuned controller, in this case IMC-PI, the IAE_d and T_s upper bound values are equivalent. Equations (6) and (8) show that a controller tuned using IMC-PI will be in the High Performance Class for $\tau_c/\theta_a \leq 1.57$. Other setpoint response tuning methods, such as IAE (setpoint) also produce PI controllers in the High Performance Class (Swanda, 1999).

Controllers in the High Performance Class also have an acceptable degree of robustness. For IMC-PI control with $\theta_a \leq \tau_c \leq 1.57\theta_a$, $3.1 \leq A_m \leq 4.0$ and $61^\circ \leq \phi_m \leq 68^\circ$ from (11) and (12), respectively. To determine if an arbitrarily tuned controller, which is inside the High Performance Class, has an acceptable degree of robustness, T_s , IAE_d , OS, A_m , and ϕ_m are determined for a wide range of PI controller settings, which includes both poorly-tuned and well-tuned controllers. Again Models 1 – 5 were considered. For each model, 2500 controller settings were simulated using all possible combinations of 50 equally spaced K_c and 50 τ_I values. The range of K_c and τ_I values spanned approximately one order of magnitude and are approximately centered on the IMC-PI settings. Table 3 shows the minimum and maximum A_m and ϕ_m values for the subset of these controller settings that are in the High Performance Class. Clearly, controllers in the High Performance Class have an acceptable degree of robustness.

If the best achievable performance of PI control is desired, than the controller should be retuned if it is outside the High Performance Class. Furthermore, determining if a controller has the best achievable performance is useful, because if this ideal limit does not meet manufacturing specifications, then retuning the PI controller will not solve the problem. In this situation, a more advanced controller, such a PID or model predictive controller, would need to be considered.

The bounds for the Excessively Sluggish and Poorly Tuned Classes were determined based on a performance-robustness argument. For many industrial applications, control loop stability over a wide operating range is more important than the best achievable performance. This is achieved by detuning the controller to trade-off performance for robustness. However, a point is reached where the robustness is excessive, and control loop performance is poor for no good reason. In typical control applications, $A_m > 10$ and $\phi_m > 80^\circ$ are not required and are indicative of an unnecessarily high degree of robustness. Performance benchmarks corresponding to $A_m = 10$ and $\phi_m = 80^\circ$ can be determined using the IMC-PI method. Substituting either for $A_m = 10$ into (13) or $\phi_m = 80^\circ$ into (14) and solving for the dimensionless settling time yields $T_s = 13.3$. From (9) the corresponding IAE_d value is 6.3. These two values are used as bounds for the Excessively Sluggish and Poorly Tuned Classes in Table 2.

Furthermore, equations (13) and (14) provide new insight into how the IMC-PI tuning method trade-offs performance with robustness. These equations can be used to determine

robustness benchmarks for the current level of performance. For example, if $T_s = 5$, the corresponding benchmark values for A_m and ϕ_m are 4.3 and 69° . If it is determined that the A_m and ϕ_m values are significantly less than the benchmarks, then the controller has a poor performance-robustness trade-off, and retuning of the controller is advisable

An overshoot value of 10% is used to distinguish between the Excessively Sluggish and Poorly Tuned Classes. A characteristic of a detuned controller is that it has little or no overshoot. If a controller is excessively detuned, then it should definitely exhibit no overshoot. Therefore, if $OS > 10\%$ and the T_s and IAE_d upper bounds are exceeded, then the controller is considered to be poorly tuned. In fact, an $OS \leq 10\%$ bound can be applied regardless of the values of the dimensionless performance indices. The responses for IMC-PI, optimal T_s , and optimal IAE_d controllers have $OS \leq 10\%$ for the five models considered.

6. CONCLUSIONS

A new methodology has been proposed to assess the performance of PI control from closed-loop response data for a single setpoint step change. The methodology is based on two new dimensionless performance indices, the dimensionless settling time and IAE values. It has been shown that benchmark values of several different closed-loop systems are independent of the process model. Thus with limited knowledge of the underlying process, the performance of PI controllers can be assessed.

New insight has been gained concerning the performance-robustness trade-off inherent in the IMC tuning method. Simple, analytical relationships have been derived which relate the dimensionless performance indices, the gain margin, and the phase margin to the IMC tuning parameter, τ_c .

The proposed methodology is also applicable to PID controllers and to non-self regulating processes (Swanda, 1999).

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Table 1. Optimal T_s and IAE_d values for PI control.

Process Model	T_s	IAE_d
1 ($\theta_a/\tau_a = 0.1$)	2.4	2.1
1 ($\theta_a/\tau_a = 0.5$)	2.3	2.0
1 ($\theta_a/\tau_a = 1.0$)	2.2	1.9
2	2.3	2.0
3	2.3	2.0
4	2.4	2.0
5	2.3	2.0

Table 2. Proposed performance classes for PI control.

Class	T_s	IAE_d	OS
High Performance	≤ 4.6	≤ 2.8	*
Excessively Sluggish	> 13.3	> 6.3	$\leq 10\%$
Poorly Tuned	> 13.3	> 6.3	$> 10\%$

* No specification required.

Table 3. The minimum and maximum A_m , and ϕ_m values for the considered PI controller settings that are in the High Performance Class.

Process Model	A_m		ϕ_m	
	min	max	min	max
1 ($\theta_a/\tau_a = 0.1$)	1.9	4.1	40	69
2	2.1	4.6	46	76
3	1.9	4.7	40	75
4	2.2	4.5	49	76
5	2.0	4.2	46	75
Average	2.0	4.4	44	74

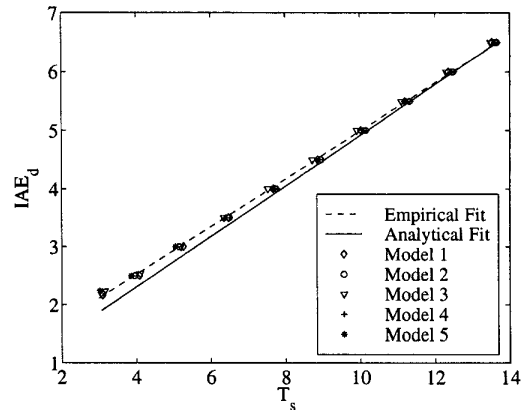


Figure 1. Comparison of the analytical and empirical relationships between T_s and IAE_d with the numerically determined values for Models 1 – 5.

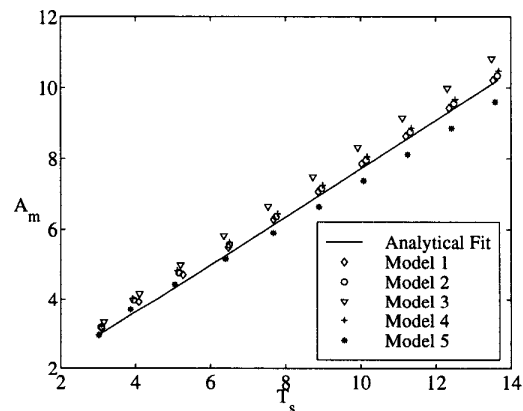


Figure 2. Comparison of the analytical and empirical relationships between T_s and A_m with the numerically determined values for Models 1 – 5.

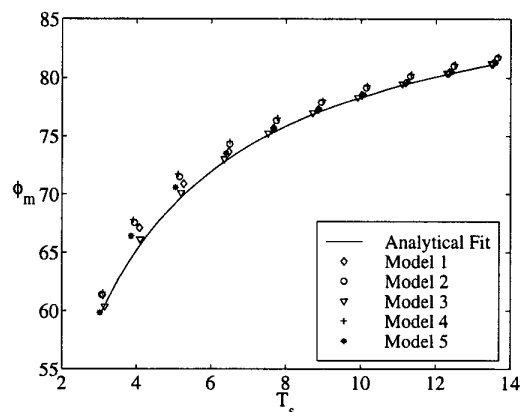


Figure 3. Comparison of the analytical and empirical relationships between T_s and ϕ_m with the numerically determined values for Models 1 – 5.