ROBUST CONTROL
OF A WASTEWATER TREATMENT SYSTEM

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Abstract

A robust controller is designed for a wastewater treatment system. Due to the periodic behavior of this process, a frequency domain approach to robust control design is used. The robust controller is compared via simulation with conventional methodologies of controlling this process.

1 Introduction

The purpose of a wastewater treatment system is to reduce the concentration of both physical and chemical contaminants to meet standards set by the Environmental Protection Agency (EPA). To achieve this objective, numerous purifying operations are necessary. One key operation involves a biological process to remove organic matter in the waste through biochemical oxidation. Aerobic biological processes employing an 'activated sludge reactor' are among the most commonly used systems. A schematic for this system is shown in Figure 1. After oxidation in an activated sludge reactor and separation, generally 95 percent of the biochemical oxygen demand (BOD5) exerted by organic matter from the inlet stream has been removed. The clarified wastewater is contained in an effluent stream from the separator and is sent for further treatment. The waste stream from the separator, which contains the remaining BOD5, is divided into a purge stream and a recycle stream. The purpose of the recycle stream is to increase the concentration of biomass in the reactor to promote oxidation of organic solids.

The development of process control strategies for this wastewater treatment system has been hampered primarily due to, 1) large uncertainty in kinetic models describing the oxidation of organic matter (or, alternatively, the growth of biomass) in the reactor and, 2) lack of adequate on-line sensors to measure the organic concentration in the wastewater and biomass in the reactor. In this paper, we will chiefly be concerned with the first problem although the control strategy we develop will require only the measurement of biomass concentration (which, in general, may be easier to measure than the concentration of organic material).

Because of the two problems mentioned above, simple control strategies which manipulate the recycle flow rate in proportion to the inlet flow rate have been employed. Recently, Tenno et al. [1] have developed a stochastic control strategy for this process. This methodology requires a priori knowledge of numerous model parameters. The control strategy does not take into account the large uncertainty typically associated with these parameters. Thus stability and performance robustness guarantees cannot be explicitly considered in controller design.

In this paper, we develop a robust controller for the activated sludge process. An example is presented in which the proposed robust controller is compared with conventional ones.

2 Process model

To develop a model of the process, a hyperbolic Michaelis-Menton relationship is used to describe the growth of biomass in the activated sludge. Employing this relationship, material balances on the substrate (organic material or 'food') and the biomass (microorganisms) are given by [2] (see Section 7 for nomenclature):

\[
\frac{dS}{dt} = \frac{Q_o S_o + Q_r S - (Q_o + Q_r)S}{Y(K_m + S)} - \frac{k_o S X V}{(K_m + S)}
\]

\[
\frac{dX}{dt} = \frac{Q_o X_o + Q_r X - (Q_o + Q_r)X}{(K_m + S)} - \frac{k_o S X V}{(K_m + S)}
\]

The parameters \(K_m, k_o, k_d, Y\), and \(Y\), used in the kinetic expressions are characterized as unknown but

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bounded. The inlet flow rate, \( Q_o \), acts as a measured disturbance. The inlet substrate concentration, \( S_o \), acts as an unmeasured disturbance. The inlet concentration of biomass, \( X_o \), will be assumed to be negligible in the remainder of this paper.

The concentration of the biomass in the recycle stream, \( X_r \), depends upon the specific type of separator used and its characteristics. We consider the separator to behave as a sedimentation vessel. Consequently, a constant ratio of output to input solids concentration is maintained. The separator behavior can therefore be modeled by

\[
\beta = \frac{X_r}{X}
\]  

(3)

Because this relationship is only an approximation to the actual behavior of the separator and assumes that the separator dynamics are negligible, the parameter \( \beta \) is characterized as unknown but bounded.

The sensor which measures the concentration of biomass in the recycle stream is modeled by a first-order transfer function,

\[
\tau_m \frac{dX_m}{dt} = X_r - X_m + n
\]  

(4)

where \( n \) is measurement noise. Alternatively, the measurement of the biomass in the activated sludge reactor, \( X_l \), can be inferred by measuring the concentration of \( \text{CO}_2 \) and \( \text{O}_2 \) gases in the reactor [1].

3 Control objectives

To obtain efficient operation, the activated sludge process must achieve good settling of the biomass in the separator and high conversion of the entering organic material, \( S_o \). The process loading factor, denoted by \( U \), serves as an indicator of both the sludge settling characteristics and the sludge activation [2]. This parameter is defined as the ratio of the mass of substrate consumed in the reactor per day to the mass of microorganisms in the reactor. For the system considered here, \( U \) (also called the F/M ratio) is given by

\[
U = \frac{Q_o(S_o - S)}{XV}
\]  

(5)

For most conventional activated sludge processes, the optimum value of \( U \) lies between 0.2-0.6 kg BOD\textsubscript{5}/kg MLSS-day, where the sludge generated has good settling velocity and high removal of BOD\textsubscript{5} in \( S_o \) is achieved. The process loading factor can thus serve as a reference signal for the control system. In the example we present later, it is desired to maintain \( U \) at a value of 0.5 kg BOD\textsubscript{5}/kg MLSS-day despite diurnal variations in \( Q_o \). A reference signal for the concentration of the biomass in the recycle stream can be formed from Equation (5) as

\[
X_{rd} = \frac{\beta(S_o - S)Q_o}{U_dV}
\]  

(6)

where \( U_d \) denotes a desired value of \( U \). Now, considering that \( Q_o \) is a measured disturbance and \( S_o - S \) is unmeasurable but reasonably constant if adequate control is maintained, a time varying reference signal can be formulated

\[
X_{rd} = k_uQ_o
\]  

(7)

Consequently, it follows that by measuring \( Q_o \) and controlling \( X_r \) about \( X_{rd} \), \( U \) can be controlled approximately about its desired value, \( U_d \). A conventional manipulated variable for the activated sludge process is the flow rate in the recycle stream, \( Q_r \).

4 Robust control design

We employ a linear framework to design a robust controller. The nonlinear system of equations describing the activated sludge process must first be expressed as a linear system. This transformation can be accomplished by bounding the nonlinear relationship by two linear ones [3]. To employ this approach, operating windows are first defined over which all future robustness results will hold. For our specific case, we define windows for \( Q_o \) \( \in \{Q_{o \text{min}}, Q_{o \text{max}}\} \), \( S \in \{S_{\text{min}}, S_{\text{max}}\} \) and \( X_r \in \{X_{r \text{min}}, X_{r \text{max}}\} \). Since we are interested in developing a relationship between the controlled variable \( X_r \) and the manipulated variable \( Q_r \), it is only necessary to consider Equation (2). Provided that the process variables stay within their respective operating windows, it can be shown that the derivative of \( X_r \) will be given by

\[
\frac{dX_r}{dt} = (C_1 + \Delta_1 R_1)X_r + (C_2 + \Delta_2 R_2)Q_r
\]  

(8)

for some \( \Delta_1 \in [-1,1], \Delta_2 \in [-1,1] \). Now let \( \alpha_1 = (C_1 + \Delta_1 R_1) \) and \( \alpha_2 = (C_2 + \Delta_2 R_2) \) which have the following minimum and maximum values

\[
\alpha_{\text{min}}^1 = \frac{k_o^{\text{min}} - q_{\text{min}}^{\text{min}} - Q_{\text{max}}}{k_m^{\text{max}} + S_{\text{min}}}
\]

\[
\alpha_{\text{min}}^2 = \frac{k_o^{\text{min}} + q_{\text{min}}^{\text{min}} - Q_{\text{max}}}{k_m^{\text{min}} + S_{\text{max}}}
\]

\[
\alpha_{\text{max}}^1 = \frac{X_{\text{min}}}{V}(\beta_{\text{min}} - 1)
\]

\[
\alpha_{\text{max}}^2 = \frac{X_{\text{max}}}{V}(\beta_{\text{max}} - 1)
\]

Consequently, we have

\[
C_1 = \frac{\alpha_{\text{max}}^1 + \alpha_{\text{min}}^2}{2}
\]  

(9)
\[ C_2 = \frac{\alpha_2^{\text{max}} + \alpha_2^{\text{min}}}{2} \quad (10) \]
\[ R_1 = \alpha_1^{\text{max}} - C_1 \quad (11) \]
\[ R_2 = \alpha_2^{\text{max}} - C_2 \quad (12) \]

The inlet concentration of biomass, \( X_a \), has been set to zero as previously discussed. The family of linear transfer functions formed by Equation (8) will be denoted by \( \Pi \).

To design a robust control system for this uncertain system, we first consider that the effect of external disturbances and set-point changes on closed-loop performance can be characterized through the sensitivity function \( \epsilon(s) \),

\[
\frac{X_{rd}(s) - X_r(s)}{G_m(s)G_d(s)D(s) - X_{rd}(s)} = \frac{X_r(s)}{ \frac{1}{1 + G_p(s)G_c(s)G_m(s)} \equiv \epsilon(s) (13) \}
\]

where the transfer functions are defined in the Nomenclature Section. The effect of measurement noise on the output signal can be characterized by the complementary sensitivity function \( \eta(s) \),

\[
\frac{X_r(s)}{X_{rd}(s) - G_m(s)N(s)} = \frac{G_p(s)G_c(s)}{1 + G_p(s)G_c(s)G_m(s)} \equiv \eta(s) (14) \]

which is related to the sensitivity function by

\[
\epsilon(s) + G_m(s)\eta(s) = 1 (15) \]

The first requirement in the design of any robust control system is to guarantee stability despite model uncertainty. The second requirement for our case is to achieve good robust tracking of the time varying reference signal \( X_{rd} \) driven by \( Q_o \). Because the behavior of wastewater treatment systems can be well characterized as periodic, it is convenient to develop and analyze robust control systems for this process in the frequency domain. We thus consider the \( H_\infty \) control problem stated as

\[
\min_{G_c} \max_{G_p} ||\epsilon_w||_{\infty} = \min_{\epsilon} \max_{G_p} \sup_{\omega} |\epsilon_w(j\omega)| (16) \]

The weighting function \( \epsilon(j\omega) \) can be chosen in accordance with the expected periodic signals of the plant. For example, one would expect that the inlet flow rate \( Q_o \) driving the reference signal \( X_{rd} \) (see Equation (7)) would be diurnal in nature. Consequently, a high weighting function can be placed near \( \pi/12 \) rad/hr to force the sensitivity function to a low value. A reasonable bandwidth for the system can also be chosen by considering that the control system should be able to attenuate variations with a period greater than 2 hours. This corresponds to choosing a bandwidth of \( \pi \) rad/hr.

In this paper, a suboptimal solution to the \( H_\infty \) control problem will be employed. Our approach will be to fix the structure of a low order controller and optimize its parameters with respect to the magnitude of the sensitivity and complementary sensitivity functions evaluated at a discrete set of frequencies. Based upon the previous discussion, a simple PI controller should be adequate for this process. The integral action will provide a large low-frequency gain for tracking and the proportional action will yield a desirable bandwidth. The PI controller is expressed by

\[
G_C(s) = k_1 + \frac{k_2}{s} (17) \]

In addition to robust stability, tracking and disturbance rejection concerns, this two mode controller should also be designed such that measurement noise is not significantly amplified. The optimization is thus formulated as a constrained multiobjective problem. We first define a vector of objective functions,

\[
x_1(k_1, k_2) = \max_{\epsilon} \epsilon(j\omega) | \]
\[
\vdots \]
\[
x_k(k_1, k_2) = \max_{\epsilon} \epsilon(j\omega_k) | \]
\[
x_{k+1}(k_1, k_2) = \max_{\epsilon} \eta(j\omega_{k+1}) | \]
\[
\vdots \]
\[
x_{k+l}(k_1, k_2) = \max_{\epsilon} \eta(j\omega_{k+l}) | \]

The tradeoff between tracking, disturbance rejection, and noise amplification can be simultaneously considered through the above formulation. These tradeoffs, however, must be considered under the constraint that robust stability is achieved which can be checked numerically.

### 5 Example

The numerical values for the kinetic parameters considered for the example are summarized in Table 1. The uncertainty for the kinetic parameters was chosen to coincide with the typical range for domestic wastewater as reported in [2]. The volume of the tank was taken as 1.5 \times 10^3 \ell. A nominal operating point was chosen based on typical operating values for the residence time, \( \theta_{\text{nom}} = V/Q_o = 8 \) hr, and for the recycle ratio, \( \theta_{\text{nom}} = Q_{r\text{nom}}/Q_o = 0.31 \). For an inlet substrate concentration, \( S_0 \), of 300 mg BODs/\ell, the outlet soluble waste concentration, \( S_{\text{nom}} \), is 6 mg BODs/\ell and concentration of biomass in the recycle
stream, $X^s_{0,om}$, is 7070 mg MLSS/l. This operating point corresponds to a value of $U_d = 0.5$ day\(^{-1}\) and $k_T = 4.7 \times 10^{-4}$ mg MLSS-hr/l\(^2\). The operating window over which our performance and robustness results will hold was selected as $Q_o \in [0.75Q^s_{0,om}, 1.25Q^s_{0,om}]$, $S \in [0, 20]$, $X_r \in [4 \times 10^4, 1 \times 10^5]$. The diurnal variation in flow rate was taken as

$$Q_o = Q^s_{0,om}(1 + 0.25 \sin \pi t/12) \quad (18)$$

The filter time constant was selected as $1/12$ hr. Measurement noise with a variance of 1200 was included in the simulations.

Table 1: Nominal values and ranges of kinetic and separator coefficients

<table>
<thead>
<tr>
<th>$k_s$ [hr(^{-1})]</th>
<th>0.0025 - 0.003</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_o$ [hr(^{-1})]</td>
<td>0.2 - 0.3</td>
</tr>
<tr>
<td>$K_m$ [mg/l]</td>
<td>90 - 60 - 120</td>
</tr>
<tr>
<td>$Y$ [mg MLSS/mg BOD(_5)]</td>
<td>0.6 - 0.5 - 0.7</td>
</tr>
<tr>
<td>$\beta$</td>
<td>4 - 3 - 5</td>
</tr>
</tbody>
</table>

A multiple objective criterion was formulated. First, it was considered desirable to make the sensitivity function small at the frequency of $\omega = \pi/12$ to achieve good tracking and at $\omega = \pi$ to achieve adequate disturbance rejection. It was also desirable to make the complementary sensitivity function small at the frequency of $\omega = 24\pi$, which corresponds to the time constant of the measurement sensor, so as not to induce severe amplification of measurement noise. The vector of performance-related objective functions was thus of dimension three.

A numerical computation was then performed to determine the set of parameters in the PI controller which guaranteed robust stability and a bandwidth of at least $\pi$. Of the members of this set, the values of $k_1 = 10,000$ and $k_2 = 12,000$ were selected to achieve a desirable tradeoff between tracking and noise sensitivity. In Figures 2 and 3, contour plots are shown depicting the variation of the objective functions with respect to $k_1$ and $k_2$. Note that the integral action has a small influence on the high frequency objective functions.

The values of the objective functions for this controller are then

$$x_1 = \max_{\delta_s \in \Omega} | \epsilon(j\pi/12) | = 0.012$$

$$x_2 = \max_{\delta_s \in \Omega} | \epsilon(j\pi) | = 0.70$$

$$x_3 = \max_{\delta_s \in \Omega} | \eta(j24\pi) | = 0.37$$

The bandwidth of the system is thus at least $\pi$. The bounds on the magnitude of $\epsilon$ and $\eta$ corresponding to this controller are shown in Figures 4 and 5, respectively. Note that some amplification of noise will take place with this control system. A readjustment of the weights can be used to redesign the controller to suppress noise if necessary.

The input/output behavior of the controlled process was then analyzed and compared with two existing industrial approaches to control this wastewater treatment system. The first control method relies on an overdesign of the process (which thereby increase capital and operating costs) to handle large variations in inlet conditions and maintain stability. That is, performance is not considered and the process is essentially run in open-loop. For our example, this method corresponds to maintaining

$$Q_r = 0.31Q^s_{0,om} \quad (19)$$

The second method uses a heuristic approach in which the recycle ratio, $Q_r/Q_o$, is maintained constant. For this example, the control law is thus

$$Q_r = 0.31Q_o \quad (20)$$

In order to incorporate the effect of model uncertainty, we chose to vary the parameters $K_m$, $k_s$, $k_o$, $Y$, and $\beta$, in a sinusoidal fashion with frequencies of $\pi/2$, $2\pi/3$, $\pi/3$, and $\pi/6$ rad/hr, respectively, and phases of 0, $4\pi/3$, $\pi/3$, and 0 radians, respectively. The amplitude was selected so that the entire range of uncertainty was covered. The inlet flow rate, $Q_o$, which drives the reference signal, was characterized by the diurnal variation of Equation (18).

The robust control strategy was then compared with the conventional control strategies of Equations (19) and (20). The response of the concentration of the biomass in the recycle stream $X_r$ to the set-point change induced by Equations (18) and (7) is shown in Figure 6. It was seen that the robust controller’s ability to track the set-point was achieved at the expense of amplification of measurement noise by a factor of 3.

The response of the concentration of soluble organic solids, $S$, for the robust control scheme is presented in Figure 7. The flow rate of the recycle stream, that is the manipulated variable for the robust control strategy, is shown in Figure 8. From these results it may be concluded that, 1) a robust controller can be made insensitive to large uncertainty/variations in model parameters for this process and, 2) a simple approach to robust control design for the activated sludge process can provide better results than control methodologies which concentrate mainly on stability and neglect dynamic performance behavior.

We note that for this particular variation of $Q_o$, the recycle flow rate did not fall below zero. If this had been the case, the robust control guarantees would no longer hold. To take this situation into account, an additional uncertainty term can be added to Equation (8) to characterize the finite range of the actuator.
6 Conclusion

A robust controller has been designed for a wastewater treatment system. The periodic behavior of this process made it convenient to design the controller in the frequency domain. Simulation results have demonstrated that a robust controller can be made insensitive to model uncertainty and can provide better set-point tracking than conventional approaches for controlling the activated sludge process.

7 Nomenclature

\begin{align*}
D(s) & \text{ Laplace transform of disturbance} \\
G_c(s) & \text{ transfer function of the controller} \\
G_d(s) & \text{ transfer function from } D(s) \text{ to } X_r(s) \\
G_m(s) & \text{ transfer function of the sensor} \\
G_p(s) & \text{ transfer function from } Q_r(s) \text{ to } X_r(s) \\
k_d & \text{ endogeneous rate constant, hr}^{-1} \\
k_o & \text{ growth rate constant, hr}^{-1} \\
k_U & \text{ proportionality factor relating } Q^* \text{ to } X_d, (\text{mg MLSS}/(\text{hr})(/\ell^2)) \\
k_1, k_2 & \text{ PI controller parameters} \\
K_m & \text{ substrate saturation constant, mg/\ell} \\
n & \text{ measurement noise, mg MLSS/\ell} \\
Q_e & \text{ effluent flow rate, \ell/hr} \\
Q_i & \text{ influent flow rate, \ell/hr} \\
Q_r & \text{ recycle flow rate, \ell/hr} \\
Q_r(s) & \text{ Laplace transform of } Q_r \\
S & \text{ substrate concentration, mg BOD5/\ell} \\
S_o & \text{ fresh feed substrate concentration, mg BOD5/\ell} \\
U & \text{ process loading factor, day}^{-1} \\
U_d & \text{ desired value of } U, \text{ day}^{-1} \\
V & \text{ volume, \ell} \\
w(\omega) & \text{ frequency domain weighting function} \\
X & \text{ biomass concentration, mg MLSS/\ell} \\
X_e & \text{ effluent biomass concentration, mg MLSS/\ell} \\
X_m & \text{ measured value of biomass concentration, mg MLSS/\ell} \\
X_r(s) & \text{ Laplace transform of } X_r \\
X_{rd}(s) & \text{ Laplace transform of } X_{rd} \\
X_{rd}(s) & \text{ Laplace transform of } X_{rd} \\
X_m(s) & \text{ Laplace transform of } X_m \\
X_o & \text{ fresh feed solids concentration, mg MLSS/\ell} \\
Y & \text{ biomass synthesis constant, mg MLSS/mg BOD5} \\
\beta & \text{ ratio of } X_r \text{ to } X \\
\epsilon(\ell) & \text{ sensitivity function} \\
\eta(\ell) & \text{ complementary sensitivity function} \\
\Pi & \text{ family of transfer functions} \\
\tau_m & \text{ time constant of sensor, hr}
\end{align*}

References


Figure 1: Schematic diagram of activated sludge process with recycle.

Figure 2: Effect of controller parameters on frequency response characteristics.
Figure 3: Effect of controller parameters on frequency response characteristics.

Figure 6: Controlled dynamic response of \( X \) for sinusoidal variation in \( X_{ad} \): solid - robust controller; dash - set-point; dotted - constant recycle ratio (Equation (19)); dash-dot - open-loop (Equation (20)).

Figure 4: Upper and lower bounds for the magnitude of the sensitivity function.

Figure 7: Dynamic response of the concentration of organic solids.

Figure 5: Upper and lower bounds for the magnitude of the complementary sensitivity function.

Figure 8: Response of \( Q_r \), the manipulated variable.