Department of Chemical Engineering — Fall, 2005
University of California, Santa Barbara

ChE 230A / ME 244A

Problem Set No. 4

Due October 20, 2005

Problem 10:

Show that the exponential form of a Fourier series representation of a function $f(x)$ over a period L,

$$f(x) = \sum_{n=-\infty}^{+\infty} c_n e^{\frac{in2\pi x}{L}}$$

can be converted into a sine/cosine Fourier series representation,

$$f(x) = a_o + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{n2\pi x}{L}\right) + b_n \sin\left(\frac{n2\pi x}{L}\right)\right] .$$

Determine how the constants c_n, a_o, a_n, and b_n are related.

Problem 11: *(Wylie, pg. 507, #s 5, 12, 13, 14)*

Find the Fourier expansions of the periodic functions whose definitions on one period are

(a) $f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & 1 < t < 3 \\ -1, & 3 < t < 4 \end{cases}$

(b) $f(t) = \cos t, \quad -\pi/2 \le t \le \pi/2$

(c) $f(t) = e^{-t}, \quad 0 < t < 1$

(d) $f(t) = t^2, \quad 0 < t < 1$

Problem 12: *(Wylie, pg. 507, #24)*

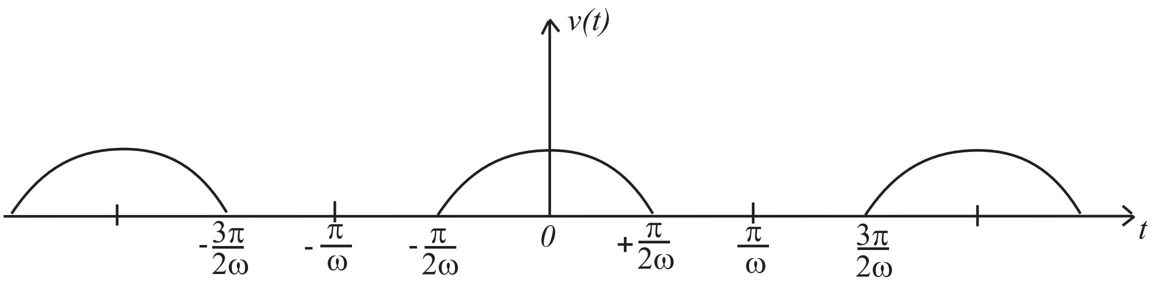
Find the Fourier expansion of the periodic function whose definition over one period is:

$$f(t) = e^t \qquad -\pi < t < \pi \quad .$$

Use your result to find the sum of the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{1+n^2}$

Problem 13: *(Kreyszig, pg. 580, #18)*

Find the Fourier series of the periodic function that is obtained by passing the voltage *v(t)* = $k \cdot \cos(100\,\pi t)$ through a half-wave rectifier, which clips the negative portion of the wave, as shown in the figure below.



Half-wave rectifier

Problem 14: *(Kreyszig, pg. 588, #s 3, 13, 19)*

Represent the following function *f(x)* by a Fourier sine series and sketch the corresponding periodic extension of *f(x)*.

(a) $f(x) = x^2 \qquad (0 < x < L)$

Represent the following functions *f(x)* by a Fourier cosine series and sketch the corresponding periodic extension of each *f(x)*.

(b) $f(x) = x^2 \qquad (0 < x < L)$

(c) $f(x) = \sin\frac{\pi x}{L} \qquad (0 < x < L)$