Inverted pendulum control

Inverted pendulum control is divided into two control problems. First is the swing up control, which allows the pendulum to reach the upright position ($\theta = 0$) and the second is the pendulum PID control around that equilibrium point (Figure 22).

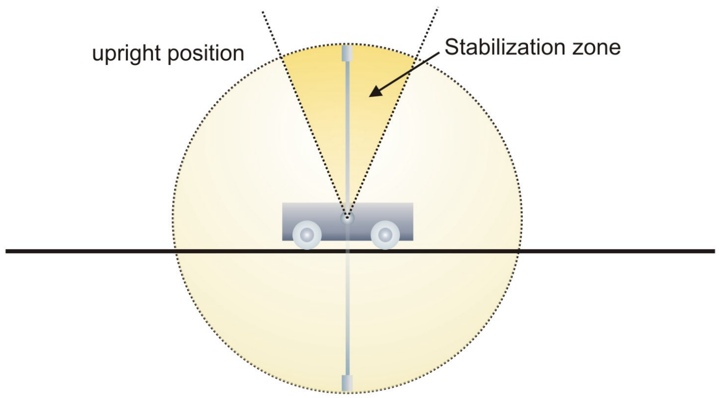

Fig. 22: Pendulum stabilization and swing up zones.

Before the inverted pendulum control aspect is discussed the swing-up strategy is considered in the following section.

Swing-up pendulum control

If the acceleration is unbounded it is possible to bring the pendulum to the upright position in a single swing. However because of safety it is better to swing up the pendulum in a robust way, which will assure that the pendulum will end up in a vertical position where $\theta = 0$. The control signal is bounded to ± 2.5 V and ± 20.0 N in terms of force value so these are the maximal values that can be transferred. The physical bound of the track has also to be considered.

In the beginning the swinging alone is considered. The goal of the control is to bring the pendulum to an upright position with as minimal angular velocity as possible. This will assure that the stabilizing controller, which takes over from the swinging controller will have an easy task. Large velocity value may cause the pendulum to over swing or cause very nervous reactions of the cart or even its' hitting of the end of the rail.

There are a lot of algorithms used for swing-up control, however their robustness is in trade-off with their time performance. In the early stages of the controller design it is better to have a robust controller and than optimize it for time duration. One of such swinging strategies comes from the pendulum kinetic and potential energy analysis and can be summed up in few rules, presented also graphically in Figure 23.

- If the θ is smaller than π/2 and 3·π/2 – pendulum is in the lower zone => apply a force until the pendulum velocity will reach $\dot{\theta} = 0$. If so, switch the direction of the force.
- If crossing upper lower zone border reverse the direction of the applied force.
- If the pendulum velocity will reach $\dot{\theta} = 0$ switch the direction of the force again. If entering the lower zone switch it once more.
- Continue force switching according to the above scheme until the angle will be in the vicinity of the vertical position by *0.2* radians for example.

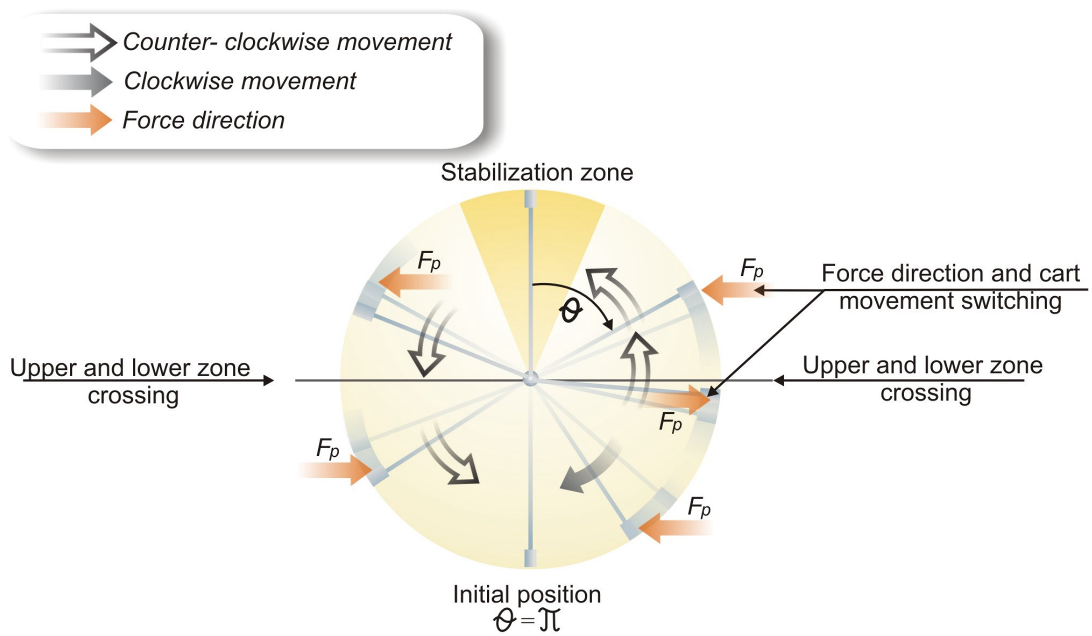

Fig. 23: Pendulum swing-up principle.

In order to inflict forces as drawn in Figure 23 the cart has to be moved the opposite way to the force arrow.

Exercise 9 – Model swing up control

Introduction

Before the swing up control will be tested in real time just as any algorithm it should be tested on a model. You are advised to design the swing up controller on your own, however the above described strategy has been designed for you in '***ModelSwingUp.mdl***'.

Task

Check how is the swinging influenced when the values of parameters (u_a – control voltage value, θ_s – absolute angle switching value) are changed. The simulation stops when the angle reaches the upper position $\theta = 0$ or $\theta = 2{\cdot}\pi$ by less than θ_s.

Exemplary results and comments

Such a strategy if programmed correctly should insure the pendulum swinging to the vertical position where $\theta = 0$. The increase of the u_a parameter will cause the swing up to be shorter, however it may cause the angular velocity to be too large at the moment of control algorithm switch to stabilization.

Exercise 10 – Real time pendulum swing up control

Introduction

For this exercise you can use the swing up control developed in exercise 9 and modify it to fit the real time application. You can also use the '***PendSwingUp.mdl***' and run the swing up control on the pendulum.

Task

Run the swing up control on the pendulum in real time. Change the parameters value (u_a – control voltage value, θ_s – absolute angle switching value) and observe the behaviour.

Exemplary results and comments

You may observe that two large or very low control voltage values cause the cart to reach the end of the track.