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ChE 152B **Winter, 2010**

Lab 1: Model Development

This lab will serve to help you determine the physical behavior of the 4-Tank System using data from the process. The overall strategy is to use a linear approximation of the process model and linear dynamic response information to generate the physical parameters to fully describe the complete nonlinear model.

1. Calibration Procedure

Before collecting any data from the 4-Tank System, you should verify the calibration of the level sensors. **This calibration may need to be repeated on a weekly basis depending on the operating conditions during the week.**

1.1 Calibration

In this section, you will learn how to calibrate the differential pressure sensors (used to measure level) on the two lower tanks. The first thing to be aware of is that the control system signals are measured in a range of -10 to +10 V. In this calibration procedure, you will match the signal reading against the actual tank height (measured by you with a ruler) to calibrate the sensor.

1. Open the Simulink model file 'calibration.mdl'

2. The two source icons determine the pump speed. Input a pump speed and start the simulation using *ctrl+T*

3. For several values of pump speed, wait for the tanks to reach a new equilibrium and record both the actual liquid level (measured with a ruler) and the potential difference (pd) readout from the Simulink window. Do this for at least 4 values in each tank at levels spanning the range of achievable liquid levels. Stop the simulation and close the Simulink window.

4. Once you have a table of pd (use *V*) vs height (use *cm*), you need to fit the data to a straight line. Hint: MATLAB and the command *polyfit* will be useful here or Excel's linear regression tool.

1.2 Operating conditions

The next thing you will need to verify is the operating condition of your system. These are dictated by the flow rate from the pumps and the valve settings. In general, you will want the lower valve settings to rest in the 90-180 region of the valve. (Note: *always adjust the lower bypass valves. The upper valve is always left fully open for normal operation.*)

Important: Be sure to record the value at which you have set the by-pass valves (corresponding to the dial position on the valve). Note that there are two scales and two corresponding pointers on the valves.

2. Model Development

Recall that the mass balances for the 4-Tank System yield the following nonlinear state-space model:

$$\frac{dx_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gx_1} + \frac{a_3}{A_1}\sqrt{2gx_3} + \frac{\gamma_1 k_1}{A_1}u_1 \tag{1}$$

$$\frac{dx_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gx_2} + \frac{a_4}{A_2}\sqrt{2gx_4} + \frac{\gamma_2 k_2}{A_2}u_2 \tag{2}$$

$$\frac{dx_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gx_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2 \tag{3}$$

$$\frac{dx_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gx_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1 \tag{4}$$

$$y_1 = x_1 \tag{5}$$

$$y_2 = x_2 \tag{6}$$

The variables are defined with respect to Figure 1.

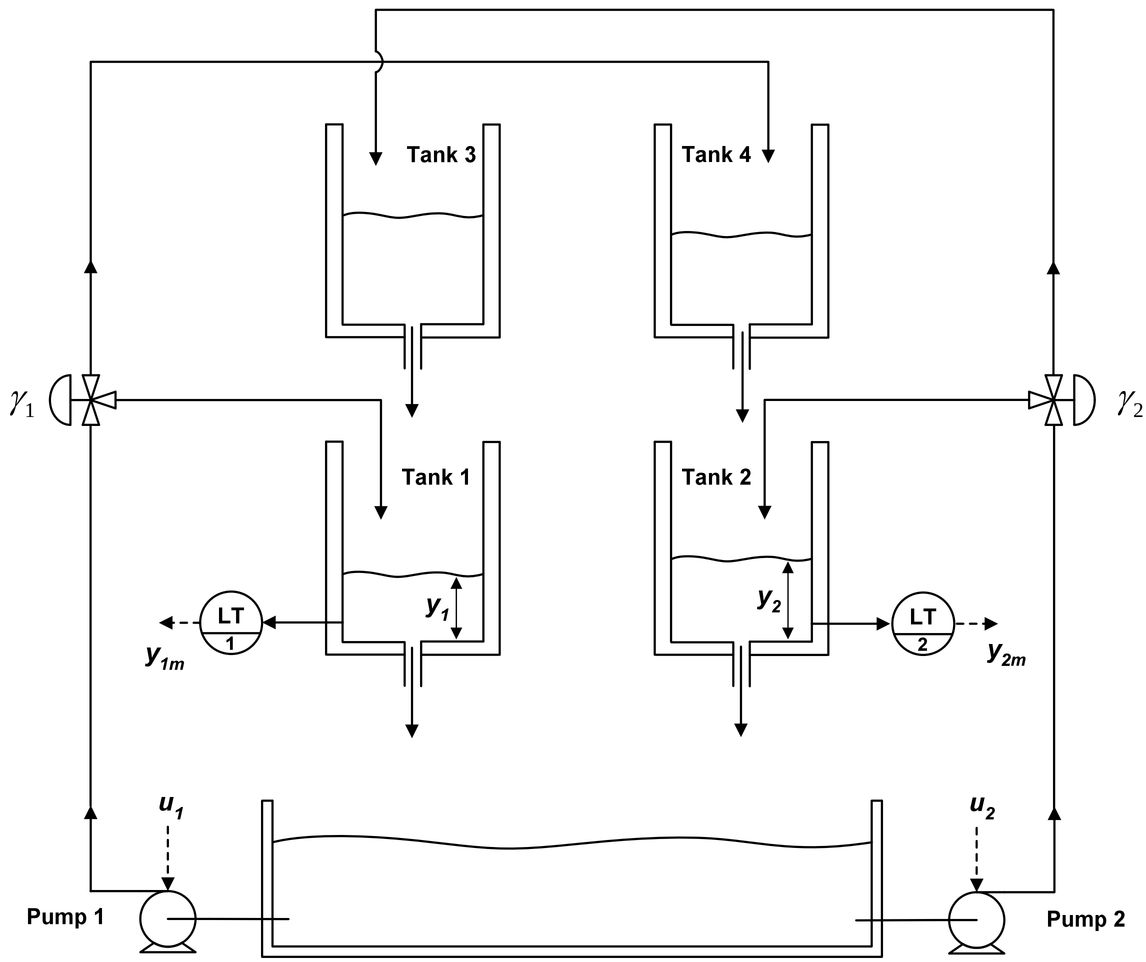

Figure 1. Schematic diagram of the 4-Tank system.

If one considers an operating point at constant pump speed $(\bar{u}_1, \bar{u}_2)$ which yields a corresponding tank height $(\bar{h}_1, \bar{h}_2)$, then linearization of the model at this operating condition yields the following transfer-function matrix for this process:

$$\mathbf{Y}(s) = \mathbf{G}(s)\mathbf{U}(s) \tag{7}$$

$$\mathbf{Y}(s) = \begin{bmatrix} Y_1(s) & Y_2(s) \end{bmatrix}^T, \qquad \mathbf{U}(s) = \begin{bmatrix} u_1(s) & u_2(s) \end{bmatrix}^T \tag{8}$$

$$\mathbf{G}(s) = \begin{bmatrix} \dfrac{\gamma_1 c_1}{\tau_1 s + 1} & \dfrac{(1-\gamma_2)c_1}{(\tau_3 s+1)(\tau_1 s+1)} \\ \dfrac{(1-\gamma_1)c_2}{(\tau_4 s+1)(\tau_2 s+1)} & \dfrac{\gamma_2 c_2}{\tau_2 s+1} \end{bmatrix} \tag{9}$$

with:

$$c_i = \frac{\tau_i k_i}{A_i} \tag{10}$$

$$\tau_i = \frac{A_i}{a_i}\sqrt{\frac{2\bar{h}_i}{g}} \tag{11}$$

The following table will prove useful in the subsequent calculations:

Symbol	State/Parameters	Value [units]
$\bar{h}_i$	Nominal level of tank *i*	TBD [*cm*]
$\bar{u}_i$	Nominal setting of pump *i*	[55; 55] [*%*]
a_i	Area of drain in tank i	TBD [cm^2]
A_i	Area of tank i	730 [cm^2]
γ_1	Ratio of flow in Tank 1 relative to flow in Tank 4	TBD
γ_2	Ratio of flow in Tank 2 relative to flow in Tank 3	TBD
k_i	Pump proportionality constants	TBD [$cm^3/\%$]
τ_i	Time constants in the linearized model	TBD [*s*]

TBD = To be determined

The task for this module will be to determine the values of the parameters labeled TBD in this table.

Once the calibration has been completed in part 1 of this lab, one can readily determine the nominal tank levels by fixing the pumps at [55%; 55%], and observing the equilibrium value of the tank level. Record these values.

3. Experimental tests to determine k_i, a_i, and γ_i

In this section, we will use a combination of steady-state and dynamic data to determine the unknown parameters in the model.

3.1 Application of calibration conditions

Open the Simulink model 'identification.mdl'. Open the yellow function blocks and enter the calibration function obtained for each sensor. **Make sure the block is converting units of V to cm.**

3.2 Steady-state testing

Adjust the pump settings using the Simulink model interface to give you a sequence of steady-state conditions corresponding to various combinations of pump settings. You are free to choose values that make sense; however, you should try to avoid any excursions larger than +/- 3-4 cm in the tank height beyond your nominal level or else the linear model approximation may break down. Be sure to record these values.

Generate at least nine different steady-state readings to build an over-determined set of equations that can be solved in a least-squares manner for the four process gains in the transfer function matrix. When you are finished stop the simulation and close the Simulink window

From these 4 gains, solve for the corresponding values of $\boldsymbol{c_i}$ and $\boldsymbol{\gamma_i}$.

3.3 Dynamic testing

From the linear transfer function matrix, it is clear that the diagonal transfer functions (pump 1 - level 1, pump 2 - level 2) are first-order. Generate a step-response experiment (again, keep your tank height within a suitable linear regime) for each pump separately, to generate data for fitting the time constants τ_1 and τ_2 in the first-order transfer functions. You should generate the time constants from a suitably transformed time domain data record. (Hint: this can also be used to cross-check the gain values).

The remaining time constants, (τ_3 and τ_4), must be estimated from a fit to the second-order response in the off-diagonal transfer functions. Use the data you already have or else generate new data to perform this identification.

Record all parameter values.

3.4 Data analysis

Comment critically on the data you have generated, noting in particular how well the model(s) fit the data with respect to both dynamic transient response as well as steady-state (gain) behavior.

3.5 Saving data

The data are exported to the workspace in a matrix called ‘identificationData’. The columns of the matrix are, in order: time (s), pump 1 (%), pump 2 (%), level 1 (cm), level 2 (cm). Save this data using the command *save yourFilename.mat identificationData*, where *yourFilename* is your choice of save name

Archival Storage of Data

Important: Be sure to back-up your data from these experiments. You may need to access the data during future experiments.
Create a folder on the desktop and name it “w10_152b_grp#”, where # is your group number. Under this directory create a subfolder and name it “Lab#” where # is the corresponding lab number.